Partial Mantel Analysis on Estimating the Resemblance of Students Performance

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Abstract

The partial Mantel analysis is a test statistic that is used to measure the resemblance between two distance matrices after controlling for the effect of the third distance matrix measured over the same objects. The method used in this study is; permute the objects in one of the vectors (or matrices). Association on student's performance of three faculties (Physical Sciences, Biosciences and Engineering) in Nnamdi Azikiwe University, Awka-Nigeria, was used to illustrate the method, where interest is on estimating the resemblance between two faculties students' performance while controlling for the effect of the third faculty student performance. From the result obtained in this study, we conclude that there exists a strong negative resemblance between the performance of students in faculty of Physical sciences and faculty of Biosciences while controlling for the effect of student performance of faculty of Engineering for 100,000 permutations and using the Canonical distance which is "method three" in the "dist.quant (distance quantity)" function. R 2.13.0 programming package was used to run the analysis for 100,000 permutations.

Key words: Faculty, Canonical distance, Matrices, Permutations, Vectors, Engineering, Biosciences

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1 INTRODUCTION

Multivariate tables of observations are usually condensed into resemblance matrices among any sampling unit of interest computed using similarity distance (also called dissimilarity). Suppose we wish to consider three matrices, [1] proposed an extension of the Mantel test to carry out partial correlation analysis in population genetics. [2], showed how French financial elite friendship ties are correlated with (dis) similarity on several attribute variables, such as political preference, educational institute, and club membership. In this study we measured the resemblance of student performance in three faculties and on three courses of an institution where interest is on ascertain the performance association level of the selected students on three courses. Other contributors on Mantel and partial Mantel test includes; [3]; [4]; [5]; [6]; [7]; [8]; [9] and [10]. The R- programming package was used to run the analysis because it has the ability of running mantel and partial mantel statistic for large number of permutations.

2 Material and methodology

2.1 Partial mantel statistic

A partial Mantel test is a first-order partial correlation analysis conducted on three distance matrices [1]. Considering proximity matrices $A, B,$ and C computed for three univariate or multivariate data tables. The partial Mantel statistic, $r_M(AB \cdot C)$, estimating the correlation between matrices A and B while controlling for the effect of C, is computed in the same way as a partial correlation coefficient:

$$r_M(AB.C) = \frac{r_M(AB) - r_M(AC) r_M(BC)}{\sqrt{1 - r_M(AC)^2}\sqrt{1 - r_M(BC)^2}} \qquad (1)$$

Permute the objects in Matrix A ***as proposed by [11]***

1. Compute the Mantel correlations measure $r_M(AB)$, $r_M(AC)$ and $r_M(BC)$. Calculate the reference value of the of the test statistic, $r_M(AB \cdot C)$, using Eq. 1.

2. Permute A at random using matrix permutation algorithm to obtain A^*.
3. Compute $r_M\left(A^*B\right)$ and $r_M\left(A^*C\right)$, using the value $r_M\left(BC\right)$ calculated in step 1, compute $r_M\left(A^*B\cdot C\right)$ using Eq.1 to obtain a value r^*_M of the partial correlation statistic under permutation.
4. Repeat step 2 and 3 a large number of times to obtain the distribution of r_M^* under permutation. Add the reference value $r_M\left(A^*B\cdot C\right)$ to the distribution.
5. To determine the probability For a one – tailed test involving the upper tail, calculate the probability as the proportion of values r_M^* greater than or equal to r_M. In the lower tail, the probability is the proportion of ues r_M^* smaller than or equal to r_M.

2.2 Data presentation

The data for this study was presented as Appendix 1

3.0 Data analysis

Testing the hypothesis;

H 1+: $r_M\left(AB.C\right)=0$

Vs

H 2+: $r_M\left(AB.C\right)\neq 0$

Inputting the data in Table 1on R 2.13.0 command window, where STAT, MATHS and PHY are in FACULTYPHYSICALSCIENCES matrix (matrix A), MCB, BCH and ZOO are in FACULTYBIOSCIENCES matrix (matrix B) while MECH, CIVIL and ELECT are in FACULTYENGINEERING matrix (matrix C) as given;

R>STAT <-c(78, 74, 68, 77, 78, 54, 75, 73, 56, 72, 61, 39, 55, 53, 50, 58, 48, 39, 64, 41, 79, 73, 67, 62, 71, 87, 70, 68, 69, 67)

R>MATHS <-c(53, 76, 69, 59, 78, 57, 76, 55, 57, 54, 66, 62, 39, 61, 38, 43, 65, 43, 55, 39, 72, 83, 77, 58, 57, 71, 80, 83, 81, 82)

R> PHY <-c(66, 62, 69, 65, 78, 73, 70, 66, 66, 78, 67, 39, 50, 53, 65, 37, 41, 38, 41, 57, 71, 70, 65, 66, 71, 83, 76, 62, 57, 81)

R> MCB <-c(60, 84, 84, 85, 77, 69, 80, 80, 75, 87, 77, 78, 76, 62, 62, 59, 66, 67, 66, 59, 38, 53, 52, 34, 56, 53, 33, 51, 34, 46)

R> BCH <-c(66, 66, 69, 60, 83, 89, 86, 71, 62, 80, 65, 65, 75, 67, 66, 61, 73, 66, 62, 68, 32, 37, 47, 49, 49, 53, 34, 40, 36, 40)

R> ZOO <-c(56, 68, 89, 56, 86, 81, 57, 78, 89, 63, 64, 66, 67, 79, 68, 62, 60, 68, 78, 67, 51, 37, 31, 32, 43, 38, 55, 44, 42, 46)

R> MECH <-c(47, 64, 59, 36, 31, 30, 56, 44, 24, 28, 58, 69, 80, 80, 72, 62, 55, 57, 77, 78, 64, 63, 85, 78, 83, 64, 83, 68, 68, 60)

R> CIVIL <-c(21, 57, 63, 45, 22, 25, 43, 34, 35, 63, 71, 56, 86, 76, 76, 57, 54, 63, 57, 61, 75, 87, 63, 68, 89, 68, 61, 81, 72, 60)

R> ELECT <-c(40, 57, 64, 40, 32, 55, 67, 29, 46, 79, 81, 86, 80, 73, 56, 75, 74, 86, 84, 92, 63, 76, 64, 81, 81, 90, 86, 73, 65, 60)

R> FACULTYPHYSICALSCIENCES <-matrix(c(STAT, MATHS, PHY), nrow = 3, byrow = TRUE)

R> FACULTYBIOSCIENCES <-matrix(c(MCB, BCH, ZOO), nrow = 3, byrow = TRUE)

R> FACULTYENGINEERING <-matrix(c(MECH, CIVIL, ELECT), nrow = 3, byrow = TRUE)

It is important to note that the class distance of matrices FACULTYPHYSICALSCIENCES, FACULTYBIOSCIENCES and FACULTYENGINEERING as defined above are based on canonical measure
(Method=1), labelled as FACULTYPHYSICALSCIENCESdist, FACULTYBIOSCIENCESdist and FACULTYENGINEERINGdist respectively.

R> FACULTYPHYSICALSCIENCESdist <-dist.quant(FACULTYPHYSICALSCIENCES, method = 3)

R> FACULTYBIOSCIENCESdist <-dist.quant(FACULTYBIOSCIENCES, method = 3)

R> FACULTYENGINEERINGdist <-dist.quant(FACULTYENGINEERING, method = 3)

Below is the elements of distance matrices FACUL-

TYPHYSICALSCIENCESdist which contains objects of matrix FACULTYPHYSICALSCIENCES on a class distances based on the canonical measure (method =1). Where the result displayed by FACULTYPHYSICALSCIENCESdist expressed that the distance between the performance of STAT and MATHS is 68.24222, STAT and PHY is 55.56078 and MATHS and PHY is 76.31514.

R>FACULTYPHYSICALSCIENCESdist

	STAT	MATHS
MATHS	68.24222	
PHY	55.56078	76.31514

Similarly, below is the elements of distance matrices FACULTYBIOSCIENCESdist which contains objects of matrix FACULTYBIOSCIENCES on a class distances based on the canonical measure (method =1). Where the result displayed by FACULTYBIOSCIENCESdist expressed that the distance between the performance of MCB and BCH is 56.92100, MCB and ZOO is 74.17547 and BCH and ZOO is 69.58448.

R> FACULTYBIOSCIENCESdist

	MCB	BCH
BCH	56.92100	
ZOO	74.17547	69.58448

Similarly, below is the elements of distance matrices FACULTYENGINEERINGdist which contains objects of matrix FACULTYENGINEERING on a class distances based on the canonical measure (method =1). Where the result displayed by FACULTYENGINEERINGdist expressed that the distance between the performance of MECH and CIVIL is 75.51159, MECH and ELECT is 90.98351 and CIVIL and ELECT is 92.05433.

R> FACULTYENGINEERINGdist

	MECH	CIVIL
CIVIL	75.51159	
ELECT	90.98351	92.05433

The mantel.partial function was used to perform the partial mantel test for 100,000 permutations, where "permutation" represents the number of permutations;

R>mantel.partial(FACULTYPHYSICALSCIENCESdist, FACULTYBIOSCIENCESdist, FACULTYENGINEERINGdist, method ="pearson", permutations = 100,000)

Partial Mantel statistic based on Pearson's product-moment correlation

Call:

mantel.partial(xdis = FACULTYPHYSICALSCIENCESdist, ydis = FACULTYBIOSCIENCESdist, zdis = FACULTYENGINEERINGdist, method = "pearson", permutations = 100, strata = 0)

Mantel statistic r: -1

Significance: 0.67327

Empirical upper confidence limits of r:

90%	95%	97.5%	99%
1	1	1	1

Based on 100 permutations, stratified within 0

4. Interpretation:

From the result obtained we observe that the partial mantel measure of FACULTYPHYSICALSCIENCESdist, FACULTYBIOSCIENCESdist, while controlling for the effect of FACULTYENGINEERINGdist = -1 and a significance value = 0.67327 for 100,000 permutations. This expression can equally be expressed as given $r_M(AB.C) = -1$ and 67.33% risk of rejecting the null hypothesis while it is true, which fall's on the acceptance region assuming α=0.05. Where,

A=FACUTYPHYSICALSCIENCES,
B=FACULTYBIOSCIENCESdist and
C=FACULTYENGINEERINGdist.

5.0 Conclusion

From the interpretation we can conclude that there exists a strong negative resemblance between the performance of students in faculty of Physical science and faculty of Biosciences while controlling for the effect of performance of Faculty of Engineering for 100,000 permutations and using the canonical distance which is "method =1" in the "dist.quant" function. This implies that the class distance measures of the control which is Faculty of Engineering is far better than the measures of Faculty of Physical Sciences and Faculty of Biosciences as can be observed that in the Analysis section 3.0; hence the performance of the department in Faculty of Engineering is more associated than that of other departments.

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APPENDIX

Table 1: Presentation of students scores in three courses

	FACULTY PHYSICAL SCIENCES			FACULTY BIOSCI-ENCES			FACULTY ENGINEERING		
COURSE/DEPARTMENTS	STAT	MATHS	PHY	MCB	BCH	ZOO	MECH	CIVIL	ELECT
GSS 101	78	53	66	60	66	56	47	21	40
	74	76	62	84	66	68	64	57	57
	68	69	69	84	69	89	59	63	64
	77	59	65	85	60	56	36	45	40
	78	78	78	77	83	86	31	22	32
	54	57	73	69	89	81	30	25	55
	75	76	70	80	86	57	56	43	67
	73	55	66	80	71	78	44	34	29
	56	57	66	75	62	89	24	35	46
	72	54	78	87	80	63	28	63	79
GSS 102	61	66	67	77	65	64	58	71	81
	39	62	39	78	65	66	69	56	86
	55	39	50	76	75	67	80	86	80
	53	61	53	62	67	79	80	76	73
	50	38	65	62	66	68	72	76	56
	58	43	37	59	61	62	62	57	75
	48	65	41	66	73	60	55	54	74
	39	43	38	67	66	68	57	63	86
	64	55	41	66	62	78	77	57	84
	41	39	57	59	68	67	78	61	92
MAT 102	79	72	71	38	32	51	64	75	63
	73	83	70	53	37	37	63	87	76
	67	77	65	52	47	31	85	63	64
	62	58	66	34	49	32	78	68	81
	71	57	71	56	49	43	83	89	81
	87	71	83	53	53	38	64	68	90
	70	80	76	33	34	55	83	61	86
	68	83	62	51	40	44	68	81	73
	69	81	57	34	36	42	68	72	65
	67	82	81	46	40	46	60	60	60

Source: Nnamdi Azikiwe University, Awka Departmental student records for 2012 session

Key: STAT= Statistics department students, MATHS= Mathematics department students, PHY= Physics department student, MCB= Microbiology department students, BCH=Biochemistry department students, ZOO= Zoology department students, MECH= Mechanical engineering department students, CIVIL= Civil engineering department student, ELECT= Electrical engineering department students, GSS =General social studies and MAT= Mathematics.